A new theory of the Aharonov-Bohm effect with a variant in which the source of the potential is outside the electronic trajectories.

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ABSTRACT. A new theory of the Aharonov-Bohm experiment, based on the calculation of the phase difference between the electronic trajectories, shows that the shifting of the interference fringes depends both on the gauge of the potential and of the location of its source with respect to the interference device. A new experiment is then suggested, in which the source of the potential is outside the electronic trajectories. The line integral of the potential along the trajectories equals zero, but the shifting ofthe fringes does not vanish.

RÉSUMÉ. Une nouvelle théorie de l'effet Aharonov-Bohm, basée sur le calcul de la différence de phase entre les trajectoires électroniques montre que l'effet dépend à la fois de la jauge du potentiel et de la position de la source par rapport au dispositif interférentiel. On propose ensuite une nouvelle expérience dans laquelle la source du potentiel est extérieure aux trajectoires. L'intégrale du potentiel le long des trajectoires est nulle, mais le déplacement des franges subsiste.

1 INTRODUCTION

The Aharonov-Bohm experiment [1], [2], [3] was conceived in order to prove the effect of a fieldless magnetic potential on electronic interferences. The idea was to introduce, between the electronic trajectories coming from two virtual coherent sources, a magnetic string, or a thin solenoid, orthogonal to the trajectories and long enough, so that the magnetic field emanating from the extremities cannot modify the electron trajectories (Fig. 1).

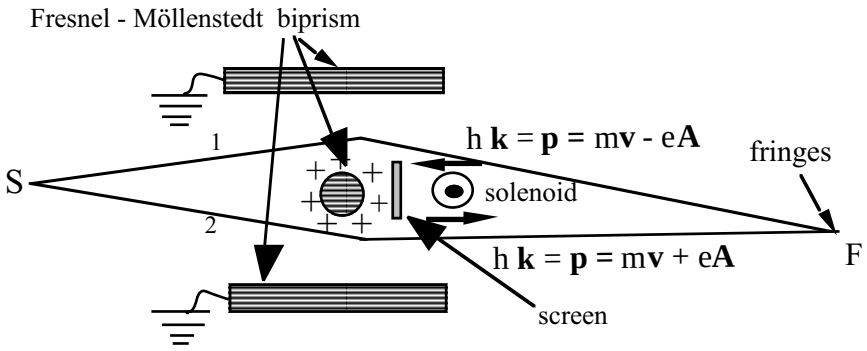

Fig. 1

Aharonov-Bohm experiment

Theoretically, in order for a magnetic flux to be trapped inside a string or a solenoid, it must be infinitely long : this is what is assumed in the calculations. But in practice, a few millimeters are sufficient because the transverse dimensions of the device are on the order of microns. As this point was contested, Tonomura [2], [3] succeeded in substituting for the rectilinear string a microscopic toroidal magnet ($\varnothing \approx 10\,\mu m$), one electron beam passing through the hole of the torus and the other passing outside, so that the magnetic lines may be regarded as beeing entirely enclosed in the magnet.

Nevertheless, in what follows, we shall restrict ourselves to an infinite magnetic string, which is sufficient for our present object, because the subtleties of Tonomura tori were invented in order to answer other arguments than those we are aiming at refuting in the present paper.

Let us give at first an intuitive interpretation of the Aharonov-Bohm experiment. Recall that the wave vector of an electron in a magnetic potential - even fieldless - is given by the de Broglie formula [5]:

$$\frac{h}{\lambda}\mathbf{n} = h\mathbf{k} = \mathbf{p} = m\mathbf{v} + e\mathbf{A} \qquad (1)$$

($\mathbf{p}$ is the Lagrange momentum). This formula is a direct consequence of the identification of the principles of Fermat and of least action : it is one of the

most reliable results of quantum mechanics. Therefore, it is *a priori* obvious that interference and diffraction phenomena will be influenced by the presence of a magnetic potential, independently of the presence or not of a magnetic field.

This phenomenon follows from a simple change of wavelength and thus a change of phase, as may be done in optics by introducing a plate of glass into a Michelson interferometer. Besides, the phenomenon is manifestly gauge dependent : if we add something to $\mathbf{A}$, whether it be a gradient or not, λ is modified. Of course, it is true even when $\mathbf{A} = 0$, i.e. for the formula $\lambda = \dfrac{h}{mv}$ in the vacuum, which is thus gauge dependent too. This fact was emphasized by de Broglie many years ago : **electron interferences are not gauge invariant.**

In the case of the Aharonov-Bohm experiment, there are additive phases on both interfering waves, and moreover they are in opposite directions, which doubles the shift of interference fringes. We furthermore give a new proof of all this.

This remarkable effect, which proves the influence of a fieldless magnetic potential on electron waves, is shocking for those who have been convinced for a century that electromagnetic potentials are only mathematical intermediate entities. And even more shocking is the fact that formula (1) imposes an electromagnetic gauge that can be measured experimentally.

The almost unanimous opinion that gauge invariance is an absolute law is so firmly fixed in prevailing thought that even distinguished physicists [4] are led to present a wrong formula for the wavelength – writing $\lambda = \dfrac{h}{mv}$ instead of formula (1), in the presence of a potential. For the same reason, Feynman managed to relate the Aharonov-Bohm effect, not with the wavelength formula (1), but with the magnetic flux trapped in the string or in the solenoid, saving in this way the gauge invariance [6], [7].

The aim of the present work is to prove that the shifting of the fringes depends on the distance from the solenoid to the experimental device and to suggest a new experiment in which the solenoid, with its magnetic flux, is outside the quadrilateral formed by the electronic trajectories, which causes the integral of $\mathbf{A}$ to vanish and makes the argument of the magnetic flux enclosed by the trajectories ineffective.

Actually, the quadrilateral itself will be removed from the calculations, rejecting to infinity the electron source and the interference fringes, which introduces negligible errors : this approximation is usual in optics.

2 A NEW THEORY OF THE AHARONOV-BOHM EFFECT

The commonly admitted theories of this effect are often complicated [2] but for the physical bases, one can read the brillant book of Tonomura [3]. Actually, the geometrical optics approximation is sufficient to answer the true question : "Where are the fringes ?". This is why we shall make use of it, assuming that we are in the case of Young slits : the other cases are topologically equivalent.

We shall define the phase of the de Broglie wave as :

$$\varphi = \frac{S}{h} \tag{2}$$

where S is the principal Hamilton function, which obeys the Hamilton-Jacobi equation :

$$2m\frac{\partial S}{\partial t} = \left(\frac{\partial S}{\partial x} + \varepsilon\frac{y}{x^2+y^2}\right)^2 + \left(\frac{\partial S}{\partial y} - \varepsilon\frac{x}{x^2+y^2}\right)^2 \tag{3}$$

where $-\varepsilon\frac{y}{x^2+y^2}$ and $\varepsilon\frac{x}{x^2+y^2}$ are the components of the potential $\varepsilon\mathbf{A}$ created by an infinite string along the Oz axis. $\varepsilon = 2\phi$ is twice the magnetic flux trapped in the string or in the solenoid (see Appendix and Fig. 2)

The electronic wave propagates from $x = -\infty$ to $x = +\infty$. The "Young slits" are on a parallel to Oy, at $\pm\frac{a}{2}$ from the point C located at $x = -b$.

The potential appearing in (3) is a gradient because :

$$-\frac{y}{x^2+y^2} = \partial_x Arctg\frac{y}{x} \;;\; \frac{x}{x^2+y^2} = \partial_y Arctg\frac{y}{x} \tag{4}$$

so that the equation (3) is easily integrated, defining :

$$\Sigma = S - \varepsilon Arctg\frac{y}{x} \tag{5}$$

which gives:

$$2m\frac{\partial\Sigma}{\partial t} = \left(\frac{\partial\Sigma}{\partial x}\right)^2 + \left(\frac{\partial\Sigma}{\partial y}\right)^2 \tag{6}$$

We choose the complete integral :

$$\Sigma = Et - \sqrt{2mE}\left(x\cos\theta_o + y\sin\theta_o\right) \tag{7}$$

Hence we get a complete integral of (3) :

$$S = Et - \sqrt{2mE}\left(x\cos\theta_o + y\sin\theta_o\right) + \varepsilon\, Arctg\frac{y}{x} \tag{8}$$

Or, in polar coordinates :

$$x = r\cos\theta\,,\; y = r\sin\theta \tag{9}$$

$$S = Et - \sqrt{2mE}\, r\cos\left(\theta - \theta_o\right) + \varepsilon\,\theta \tag{10}$$

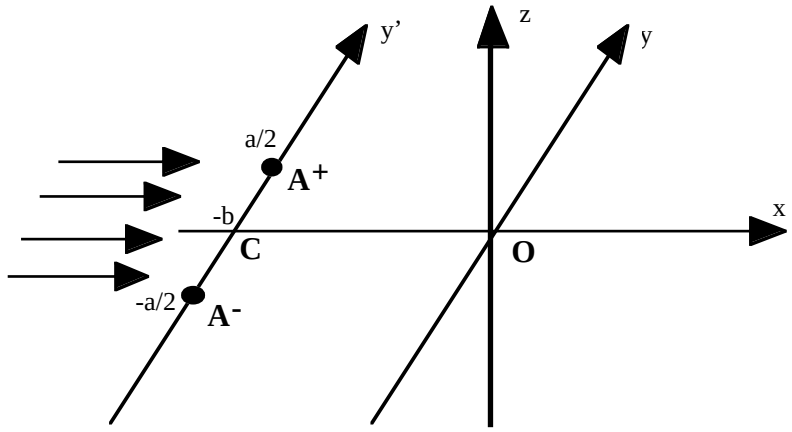

Fig. 2
Aharonov-Bohm scheme

The motion of the electron is given by the Jacobi theorem :

$$\frac{\partial S}{\partial \theta_o} = Const; \frac{\partial S}{\partial E} = Const \tag{11}$$

The trajectories are the *rays of the wave* :

$$\frac{\partial S}{\partial \theta_o} = \sqrt{2mE}\left(x\sin\theta_o - y\cos\theta_o\right) = \mu \tag{12}$$

The motion along the rays is given by :

$$\frac{\partial S}{\partial E} = t - \sqrt{\frac{m}{2E}}\left(x\cos\theta_o + y\sin\theta_o\right) = t_o \tag{13}$$

that is to say, with $E = \frac{1}{2}mv^2$:

$$x\cos\theta_o + y\sin\theta_o = v\left(t - t_o\right) \tag{14}$$

We see that the *rays* (electron trajectories) defined by (12) are *orthogonal to the moving planes* (13), but they are not orthogonal to the equal phase surfaces (8), (10) except far away from the magnetic string $\left(x \to \infty\right)$ where the ε potential term becomes negligible.

Therefore, despite the presence of the potential, *the electronic trajectories remain rectilinear and are not deviated because there is no magnetic field.* The velocity $v = Const$ is the one of the incident electrons because of the conservation of energy.

But the diffraction of the waves through the holes $\mathbf{A}^+$ and $\mathbf{A}^-$ creates, for the electron trajectories, an interval of possible angles θ_o, among which are the *angles of the interference fringes, modified by the magnetic potential*.

So there is no deviation of the electrons, but only a deviation of the angles of phase synchronization between the waves issued from A^+ and A^- because a fieldless potential can only change the phases, not the trajectories.

This is the Aharonov-Bohm effect that we now have to calculate.

Let us first look at the *orthogonal lines to the equal phase surfaces* S : they are enveloped by the *Lagrange momenta* i.e. by the de *Broglie wave-vectors*, in accordance with the formula (1), while the rays (12) are the *impulse* lines m**v**.

The momenta are :

$$\begin{aligned} p_x &= -\frac{\partial S}{\partial x} = \sqrt{2mE}\cos\theta_o - \varepsilon\frac{y}{x^2+y^2} \\ p_y &= -\frac{\partial S}{\partial y} = \sqrt{2mE}\sin\theta_o + \varepsilon\frac{x}{x^2+y^2} \end{aligned} \tag{15}$$

Hence the equation :

$$\frac{dx}{\sqrt{2mE}\,\cos\theta_o - \varepsilon\dfrac{y}{x^2+y^2}} = \frac{dy}{\sqrt{2mE}\,\sin\theta_o + \varepsilon\dfrac{x}{x^2+y^2}} \tag{16}$$

The integration is obvious thanks to the integral combinations $xdx + ydy$ and $xdy - ydx$. In polar coordinates, we find :

$$r\sin(\theta-\theta_o) - \Lambda \log\frac{r}{\Lambda} = c \quad (= Const), \quad \Lambda = \frac{\varepsilon}{\sqrt{2mE}} \qquad (17)$$

and in Cartesian coordinates :

$$y\cos\theta_o - x\sin\theta_o - \Lambda\log\frac{\sqrt{x^2+y^2}}{\Lambda} = c \qquad (18)$$

Comparing with (12), one can see that the orthogonal lines to the phase planes become parallel to the rays, far from the magnetic string. It is worth noting that phase orthogonal lines (17) or (18), and the phase velocity $V = \frac{h\nu}{p}$ (which we cannot calculate here because the frequency is correct only in relativity) depends on the potential through the momentum **p ;** but this is not the case for the electron trajectories (12) and for the electron velocity in (14).

In other words, the electrons (i.e. energy) do not follow the phase propagation, neither in velocity, nor in trajectory. The same happens in crystal optics : the phase propagation depends on the *inductions* (that is on the polarization of the medium), while the propagation of energy is given by the Poynting vector, which is only defined by *fields* and does not depend on the polarization [4].

The shifting of interference fringes.

Let us consider a plane wave propagating along Ox $(\theta_o = 0)$.The holes $\mathbf{A}^+$ and $\mathbf{A}^-$ will emit in the half space $x > 0$ two waves S^+ and S^-. According to (8), we have :

$$S^{+} = Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2}\right)\theta_o\right] + \varepsilon\, Arctg\frac{y}{x}$$
$$S^{-} = Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2}\right)\theta_o\right] + \varepsilon\, Arctg\frac{y}{x} \qquad (19)$$

where we have taken into account the smallness of θ_o : $\cos\theta_o \approx 1$, $\sin\theta_o \approx \theta_o$. Let us now suppose that $t = 0$ when $x = -b$, and let us write :

$$\xi = Arctg\frac{a}{2b} \qquad (20)$$

The initial waves S^{+} and S^{-}, in $\mathbf{A}^{+}$ and $\mathbf{A}^{-}$ $\left(x = -b, y = \pm\frac{a}{2}\right)$, are :

$$S_o^{+} = -\varepsilon\xi,\ S_o^{+} = +\varepsilon\xi \qquad (21)$$

Now, let us note that, in all the known experiments, the magnetic string, or the solenoid, was very close to $\mathbf{A}^{+}$and $\mathbf{A}^{-}$. The authors say : « in the shadow » of the electrostatic fiber of the Möllenstedt biprism [2], as it is shown on the Fig. 1. Therefore, in $\mathbf{A}^{+}$and $\mathbf{A}^{-}$, the distance b is very small and $\xi \propto \frac{\pi}{2}$, so that $S_o^{-} = +\varepsilon\frac{\pi}{2}$, $S_o^{+} = -\varepsilon\frac{\pi}{2}$, .

Therefore, we see that in $\mathbf{A}^{+}$and $\mathbf{A}^{-}$, at the beginning of the trajectories, the phases defined by (21) depend on the potential exclusively through the value of ε. On the contrary, at the other end of the trajectories, on the interference fringes, far from $\mathbf{A}^{+}$ and $\mathbf{A}^{-}$, the distance is of the order of $15cm$, while $a, b \propto 10^{-4}cm$, which justifies the approximation of parallel trajectories for the waves $S^{+}$and S^{-}in the vicinity of the fringes.

Close to the fringes, the term $\varepsilon\theta$, in (8) and (10), has practically the same value for S^{+} and S^{-} : θ is very small and $\varepsilon\delta\theta$ would be of the third order, so it disappears from (19). In other words, on the fringes,

contrary to the origin, the potential has no more influence. Finally, according to (19) and (20), the phase difference respectively undergone by the two waves propagating from $\mathbf{A}^+$ and $\mathbf{A}^-$ to the interference field will be defined by the quantities :

$$\begin{aligned} S^+ - S_o^+ &= Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2} \right)\theta_o \right] + \varepsilon\xi \\ S^- - S_o^- &= Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2} \right)\theta_o \right] - \varepsilon\xi \end{aligned} \tag{22}$$

Introducing the wavelengh $\lambda = \frac{h}{\sqrt{2mE}}$, the phase difference between the two waves will be :

$$\Delta\varphi = \frac{\Delta S}{h} = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon\xi}{h} \tag{23}$$

The first term gives the standard Young fringes, *the second one is the Aharonov-Bohm effect*. The formula (23) is not exactly in accordance with the classical theory because of the angle ξ which is absent from the classical one. ξ is half the angle under which the Young slits are seen from the solenoid.

The presence of ξ entails a dependence of the effect on the position of the string, which is in principle experimentally testable : according to (20) **the effect must decrease when the distance b increases.**

3 A NEW EXPERIMENT

We shall now suggest an experiment inspired by that of Aharonov-Bohm, but which is such that **the circular integral along the electron trajectories equals zero an thus cannot have any relation with the fringe shift**. This experiment was already suggested in [6] but as an intuitive argument. Here we give the exact calculation.

The idea is to substitute the magnetic string included between the electronic trajectories by *two strings on both sides* (Fig. 3 and 4). In

principle, one string would be enough, but we shall see that the effect is smaller than Aharonov-Bohm's, so that it is useful to double it. Owing to the new position of strings, the magnetic flux through the closed line of trajectories will be equal to zero because the potential is still a gradient and its source is outside. The effect remains, but the problem of gauge invariance is clearly irrelevant.

The Hamilton-Jacobi equation becomes here :

$$2m\frac{\partial S}{\partial t}=\left[\frac{\partial S}{\partial x}+\varepsilon\left(\frac{y-c}{x^2+(y-c)^2}+\frac{y+c}{x^2+(y+c)^2}\right)\right]+$$
$$+\left[\frac{\partial S}{\partial y}-\varepsilon\left(\frac{x}{x^2+(y-c)^2}+\frac{x}{x^2+(y+c)^2}\right)\right] \tag{24}$$

We see that, according to Fig. 3 and 4, the magnetic strings are parallel to Oz, and cut the plane xOz in two points at a distance c from Oz. We suppose :

$$c>\frac{a}{2} \tag{25}$$

in order to put the strings outside the trajectories.

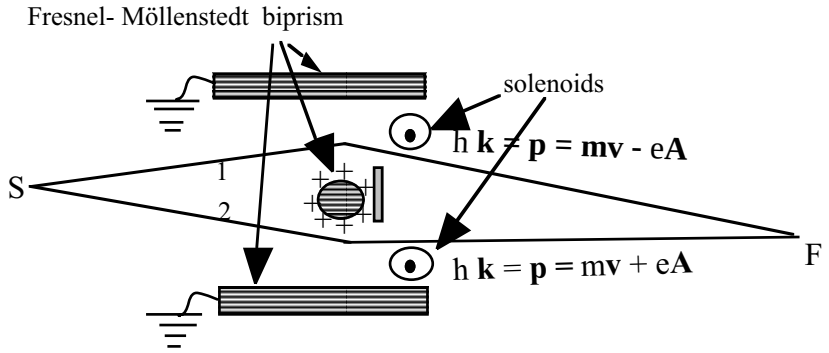

Fig. 3

New experiment

Paralleling the relations (4), we now have :

$$-\left(\frac{y-c}{x^2+(y-c)^2}+\frac{y+c}{x^2+(y+c)^2}\right)=\frac{\partial}{\partial x}Arctg\frac{y-c}{x}+\frac{\partial}{\partial x}Arctg\frac{y+c}{x}$$
$$\left(\frac{x}{x^2+(y-c)^2}+\frac{x}{x^2+(y+c)^2}\right)=\frac{\partial}{\partial y}Arctg\frac{y-c}{x}+\frac{\partial}{\partial y}Arctg\frac{y+c}{x} \quad (26)$$

And in analogy with (24) :

$$\Sigma = S-\varepsilon\left(Arctg\frac{y-c}{x}+Arctg\frac{y+c}{x}\right) \quad (27)$$

Introducing (27) in (24), we get the equation (6) again, with the complete integral (7), and finally a complete integral of (24), analogous to (8) :

$$S = Et-\sqrt{2mE}\left(x\cos\theta_o+y\sin\theta_o\right)+$$
$$+\varepsilon\left(Arctg\frac{y-c}{x}+Arctg\frac{y+c}{x}\right) \quad (28)$$

We shall not repeat the whole preceding theory. The most important thing is to note that the electron trajectories are the same straight lines as before, for the same reason : the absence of magnetic field. We find equations (12), (13), (14) again, for the wave rays. The Lagrange momenta (de Broglie wave vectors, up to a factor h) are now :

$$p_x=-\frac{\partial S}{\partial x}=\sqrt{2mE}\cos\theta_o-\varepsilon\left(\frac{y-c}{x^2+(y-c)^2}+\frac{y+c}{x^2+(y+c)^2}\right)$$
$$p_y=-\frac{\partial S}{\partial y}=\sqrt{2mE}\sin\theta_o+\varepsilon\left(\frac{x}{x^2+(y-c)^2}+\frac{x}{x^2+(y+c)^2}\right) \quad (29)$$

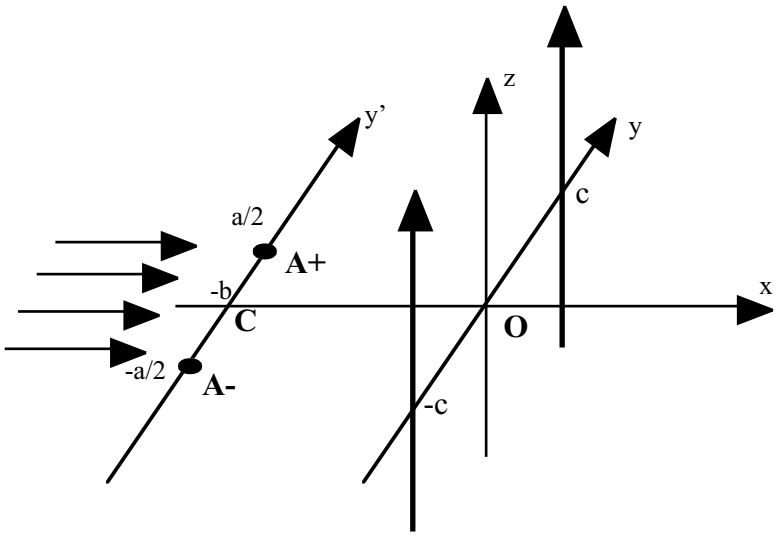

Fig. 4
New experiment scheme

The equations of the orthogonal lines of phase would be useless for the prediction of the physical effect : it was interesting to perform the integration only once, on the example (16), in order to show the difference between rays and phase lines.

The shifting of interference fringes.

Let us look once more at a plane wave coming from $x = -\infty$ to the plane $x = -b$, and diffracting through the holes $\mathbf{A}^+$ and $\mathbf{A}^-$. The angle θ_o is small again and we have, owing to (28) and in analogy with (19), two waves :

$$S^{\pm} = Et - \sqrt{2mE}\left[x + b + \left(y \mp \frac{a}{2}\right)\theta_o\right] + \\ + \varepsilon\left(Arctg\frac{y-c}{x} + Arctg\frac{y+c}{x}\right) \tag{30}$$

For we have, up to a common constant additive term :

$$S_o^+ = \varepsilon(\eta - \zeta),\ S_o^- = -\varepsilon(\eta - \zeta) \tag{31}$$

with the definitions :

$$\eta = Arctg\frac{c - \frac{a}{2}}{b};\ \zeta = Arctg\frac{c + \frac{a}{2}}{b} \tag{32}$$

Disregarding, as in (19), the small terms corresponding to the potential near the interference field (great values of x), we find the analogue of (22) for the phase differences for the waves coming from $\mathbf{A}^+$ and $\mathbf{A}^-$:

$$S^+ - S_o^+ = Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2}\right)\theta_o\right] - \varepsilon(\eta - \zeta) \\ S^- - S_o^- = Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2}\right)\theta_o\right] + \varepsilon(\eta - \zeta) \tag{33}$$

Introducing the wavelength $\lambda = \frac{h}{\sqrt{2mE}}$, we can deduce the phase difference between the two waves, just as in (23) :

$$\Delta\varphi = \frac{\Delta S}{h} = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon}{h}(\zeta - \eta) \tag{34}$$

We again find a first term, corresponding to the Young interferences, and *a second one analogous to the Aharonov-Bohm effect*. This term is smaller, for the obvious reason that each magnetic string produces a shift on the nearest trajectory, but unfortunately it also produces a shift on the other

one, and this second shift is in the same direction as the first one because both trajectories are on the same side of the string, whereas they were on opposite sides in the case of Aharonov-Bohm, so that the phase shifts on the trajectories were opposite too. This is why we find now, instead of a factor ξ, the difference $(\zeta - \eta)$, with $\eta, \zeta > 0$ because we have chosen $c > \frac{a}{2}$ in order for the string to be outside the trajectories. Nevertheless, the first shift dominates because the second trajectory is farther from the string than the first one, so that the effect does exist. And since we have two strings, the effect is doubled : hence the factor two before ε in (34).

Let us take, as an example, $c = a$. Then we have :

$$\eta = Arctg\frac{a}{2b}, \zeta = Arctg\frac{3a}{2b} \Rightarrow \max(\zeta - \eta) = 0,52 \qquad (35)$$

The maximum value of $(\zeta - \eta)$ is obtained for $b = a\frac{\sqrt{3}}{2}$. Comparing the maximum value of the Aharonov-Bohm shift in (20) : $\xi \propto \frac{\pi}{2} \approx 1,57$ with the maximum shift in (34) : $(\zeta - \eta) \approx 0,52$, we see that the effect predicted here is three times smaller. But this is not important because the aim was not to give another proof of the interference shift due to a fieldless potential (the Aharonov-Bohm proof is excellent), but to prove that an effect of the same type can be obtained with an experiment which cannot be explained in terms of a line integral which here obviously vanishes.

4 THE QUESTION OF GAUGE INVARIANCE.

There is only one problem in an interference phenomenon : **where are the fringes ?** And the answer is given by the phase difference between two waves coming from two coherent sources.

Curiously, the calculation of this phase difference is at the basis of all the interference phenomena except the Aharonov-Bohm effect ! **The interference is taken for granted and the only question is to find the shift without damaging the gauge invariance**. This is why the circular integral of $\mathbf{A}$ plays the central role. But circular integral cannot give the interfringe.

Therefore, the phenomenon is calculated in two parts : a) The « free » interference without potential. b) The shift due to the potential, considered separately and which is absent from the calculation of the phase differences, thus forgetting the geometry of the experiment. It is for this reason that the location of the solenoid and the form under which the potential enters the expression of the phase are forgotten.

If there is something new in the present paper, it is precisely an attempt to come back to the old problems and methods of interference phenomena owing to a simple calculation of phases.

Now we shall go back to the phases given by formulae (30), which include the case of (19), adding in an arbitrary gauge term $f(x, y)$. We find :

$$\begin{aligned} S^{+} &= Et - \sqrt{2mE}\left[x + b + \left(y - \frac{a}{2}\right)\theta_o\right] \\ &+\varepsilon\left(Arctg\frac{y-c}{x} + Arctg\frac{y+c}{x} + f(x, y+c)\right) \\ S^{-} &= Et - \sqrt{2mE}\left[x + b + \left(y + \frac{a}{2}\right)\theta_o\right] \\ &+\varepsilon\left(Arctg\frac{y-c}{x} + Arctg\frac{y+c}{x} + f(x, y-c)\right) \end{aligned} \tag{36}$$

Close to the slits, we have, generalizing (31) :

$$\begin{aligned} S_o^{+} &= \varepsilon\left[\eta - \zeta + f\left(\frac{a}{2} - c\right) + f\left(\frac{a}{2} + c\right)\right] \\ S_o^{-} &= \varepsilon\left[-(\eta - \zeta) + f\left\{-\left(\frac{a}{2} + c\right)\right\} + f\left\{-\left(\frac{a}{2} - c\right)\right\}\right] \end{aligned} \tag{37}$$

and the phase difference (34) becomes :

$$\Delta\varphi = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon}{h}(\eta - \zeta) + \frac{2\varepsilon}{h}\left[\begin{array}{l} f\left(\frac{a}{2} - c\right) + f\left(\frac{a}{2} + c\right) \\ -f\left\{-\left(\frac{a}{2} + c\right)\right\} - f\left\{-\left(\frac{a}{2} - c\right)\right\} \end{array}\right] \quad (38)$$

Clearly, except if $f(x, y)$ is even in y, **the phase difference is modified and that the phenomenon is not gauge invariant.**

5 APPENDIX. The magnetic potential of an infinetely thin and long solenoid or an infinite magnetic string.

We start from a classical formula in electromagnetism [8], [9], expressing the vector potential created by a magnetic dipole μ at a distance l :

$$\mathbf{A} = \frac{\mu \times \mathbf{l}}{l^3} \quad (39)$$

In a point P, the potential is equal to :

$$\mathbf{A} = \phi \int_{-\infty}^{+\infty} \frac{\mathbf{dZ} \times \mathbf{l}}{l^3} = \phi \int_{-\infty}^{+\infty} \frac{\mathbf{dZ} \times \mathbf{MP}}{|\mathbf{MP}|^3} \quad (40)$$

ϕ is the magnetic flux trapped in the string or in the solenoid, and :

$$\begin{aligned} \mathbf{MP}^2 &= \mathbf{l}^2 = x^2 + y^2 + (z - Z)^2 \\ \mathbf{dZ} \times \mathbf{MP} &= \{-y\,dZ,\ x\,dZ,\ 0\} \end{aligned} \quad (41)$$

Now we get from (40) :

$$A_x = -\phi y \int_{-\infty}^{+\infty} \frac{dz}{\left[x^2 + y^2 + (z - Z)^2\right]^{3/2}}$$
$$A_y = \phi x \int_{-\infty}^{+\infty} \frac{dz}{\left[x^2 + y^2 + (z - Z)^2\right]^{3/2}}; A_z = 0 \quad (42)$$

and given that :

$$\int_{-\infty}^{+\infty} \frac{dz}{\left[x^2 + y^2 + (z - Z)^2\right]^{3/2}} = \frac{2}{x^2 + y^2} \quad (43)$$

we find :

$$A_x = -2\phi \frac{y}{x^2 + y^2}; A_y = 2\phi \frac{x}{x^2 + y^2}; A_z = 0 \quad (44)$$

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[1] The book of Akira Tonomura, written in non-technical terms, is in principle, a popular book, but it is clear and profound and I highly appreciate it, even if I can disagree with him on some points concerning gauge invariance.